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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A MODEL FOR SCHEDULING DELIVERIES OF
REPAIR PARTS TO
MULTIPLE PRODUCTION LINES AT A NARF

by

Terry H. Darton

June 1984

Thesis Advisor: A.W. McMasters

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A Model for Scheduling Deliveries of Repair Parts to Multiple Production Lines at a NARF

by

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Lieutenant Commander, United States Navy
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I. INTRODUCTION

A. BACKGROUND

In October, 1979, Naval Air Rework Facility (NARF) Alameda was the first such facility to have its wholesale supply support provided by a Naval Supply Center (NSC) Oakland. This was the result of a recommendation from the DoD Material Distribution Study that wholesale supply support of NARF's near the NSC's at Oakland, San Diego, and Norfolk be provided by the NSC's instead of by the Naval Air Stations [Ref. 1].

The question of providing supply support for the local NARF, with no degradation of that support, became a very real concern to the NSC's. The centralization of materials at the supply centers has increased the distances to these wholesale customers; therby escalating the costs associated with moving material. Not only does the transportation cost increase, but more importantly, delays are created by extended travel time and longer picking times. These delays cost money since components may wait longer for parts. Maintenance time must be used to gather components awaiting parts and store them together until required parts are received. Because of this, valuable production resources may be diverted.

Grant [Ref. 2] attempted to quantify the production delay costs when repair parts are not immediately available when

required. He was unable to establish a firm relationship between delivery times and delay cost due to the lack of historical data at the NARF. He did determine that the slowest delivery sets the pace of the repair action and should be used to determine the production delay costs.

Under the assumption that such delay costs will eventually be quantified, McMasters [Ref. 3] developed three direct delivery models as a first step in determining the best way to support the NARF. Each alternative considered involved a single production (overhaul) line placing a single demand, at any instant of time, on the supply system. Each demand was for a repair part required by a component being reworked. The demand for this part was considered as a Bernoulli trial with a fixed known probability of demand (p) for each component inducted.

Due to the analytic complexities of McMasters' models no closed form optimization formulas were possible. Therefore, the expected total costs were used in calculating the optimal delivery schedules. Davidson [Ref. 4] performed a parametric analysis on these three models concentrating primarily on the probability of demand and production line delay costs. She discovered that there was little difference in the optimal expected costs for each model. The most practical alternative appeared to be to make scheduled deliveries.

Minimization of the expected costs, where the total cost was the sum of transportation costs and delay costs, was the goal of the work done by McMasters and Davidson. The

transportation cost was defined to be a fixed charge per delivery. Delay cost was a fixed charge per component per unit of time delayed after it was identified as needed for a given component's overhaul.

B. PURPOSE

A key assumption of the McMasters-Davidson work was that only one production line created the demand for a given repair part. Since, in reality, more than one line might need a given part, this assumption should be relaxed and the effects of that relaxation investigated. Therefore, this thesis will extend their scheduled delivery model to the consideration of multiple production lines.

C. PREVIEW

Chapter II presents the assumptions and the derivation of the formulas describing the expected costs per period for any number of production lines needing a given part. Chapter III then follows the procedure of Davidson and determines the optimal delivery schedule for a variety of cost and probability parameter values and numbers of production lines needing the part. Chapter IV will summarize the findings of the thesis and present conclusions and recommendations for further areas to be investigated.

II. MODEL FORMULATION

A. INTRODUCTION

A NARF inducts components for repair and, as the first step of the process, identifies those parts which need to be replaced. Requisitions are then prepared and transmitted to the supply system. That system picks the item from inventory and delivers it to the NARF. Upon receipt of the required parts from the supply system, the repair actions are completed and the item is returned to the system, either for immediate installation or as a shelf spare.

All of the earlier studies considered alternative transportation systems for delivering a given requisitioned repair part from the central stock point to the customer. They also attempted to minimize the sum of expected transportation costs and expected customer delay costs. McMasters [Ref. 3] considered two basic direct delivery scheduling schemes, scheduled delivery and unscheduled delivery. Under scheduled delivery, a delivery truck would be dispatched to the NARF every N periods of time and would deliver all requisitions that had been submitted up to that time by the NARF. Under unscheduled delivery the truck would make a delivery as soon as K requisitions had been received. Davidson found that the minimum expected total costs of delivery and delay were essentially the same for either scheme. This then allows a selection of the scheme which is the most desirable from other

aspects of the problem. Scheduled delivery is considered to be more desirable since it is easier for the NSC to plan and provides the NARF with a known delivery time. Scheduled delivery also represents the way many stock points presently operate their local delivery systems.

B. ASSUMPTIONS MADE IN GENERALIZING FROM PREVIOUS MODELS

In generalizing to a multiple production line system, some assumptions from the earlier model must be modified. McMasters' model referred to a time period as "the time between component inductions on the production line." For the multiple production line environment, different production lines may have different periods between inductions. However, within a group of similar lines, such as aircraft engines, the induction quantities for a given quarter have been observed to be fairly close in value. In addition, since such schedules are negotiated ahead of time for a given quarter at workload planning conferences, it is possible to obtain identical schedules for several lines. Thus, this thesis will assume equal length time periods for all production lines.

As in the case of the earlier model, transportation costs will be considered as a fixed charge per round trip from the supply center to the receiving facility. This is not an unreasonable assumption since all parts for similar lines

are usually received at a central point at the NARF and then dispersed internally to the various production lines.

We will also retain the assumption that Chambers [Ref. 5] made in his model. It was that all requirements were homogeneous; that is, no consideration was given as to priority of individual production lines over other production lines.

Delay costs are assumed to be different for each production line. We will consider only time dependent delays. The fixed costs associated with a delay, which includes the cost of removing the component from the production line and gathering associated parts, will not be considered.

The probability of demand for a given repair part by a given line is assumed to be known in advance. However, these probability values will be assumed to be different for each line.

C. DETERMINISTIC DEMAND

If a demand from a customer occurs once every time period with certainty, it is said to be a deterministic demand. Let C_T be the cost of one round trip from a supply center to the NARF. If a truck is dispatched every time a demand is received and processed, the cost of delivery for each unit is C_T. If, on the other hand, the truck waits until k units have been demanded and processed, the average delivery cost per unit is

$$C_T/k \cdot \qquad (2.1)$$

This is also the average delivery cost per period. If the truck waits until it is full (suppose it has a capacity of n units) then the delivery cost per period is minimized at

$$C_T/n \cdot \qquad (2.2)$$

While the k units are accumulating, they are creating delay costs for the NARF. If the truck waits for k units to be accumulated and the delay cost for one unit for one time period is C_D, the total average delay cost per period is

$$C_D(k-1)/2 \cdot \qquad (2.3)$$

To verify Equation (2.3), assume one unit is needed every time period. If the truck waits for k units to accumulate it will not leave until (k-1) time periods after the first demand. During this time the units ordered but not delivered have caused delay. Specifically, the first unit, ordered in the first period, will be delayed (k-1) periods; the second unit, ordered in the second time period, will be delayed (k-2) periods; etc. Only the kth unit ordered in period k will have no delay. The total waiting time in periods then is

$$(k-1) + (k-2) + (k-3) + \ldots + 1 + 0 , \qquad (2.4)$$

which can be written as $k(k-1)/2$. When this is multiplied by the delay cost per period, the result is the total delay

cost

$$C_D(k-1)k/2 \ . \tag{2.5}$$

The average delay cost per period is obtained by dividing by the number of units, k.

$$C_D(k-1)/2 \tag{2.6}$$

By adding the average shipping cost and delay costs per period, the total average cost for k units becomes

$$TAC(k) = C_T/k + C_D(k-1)/2 \ . \tag{2.7}$$

TAC(k) is a discrete function since k must be integer valued. As a consequence, minimization requires that finite differences be used. The optimum k is that value such that

$$TAC(k-1) > TAC(k) \leq TAC(k+1) \tag{2.8}$$

or equivalently the largest k such that

$$TAC(k) - TAC(k-1) < 0 \ ; \tag{2.9}$$

or the smallest k such that

$$TAC(k) - TAC(k+1) \leq 0 \ . \tag{2.10}$$

Using Equation (2.7), inequality (2.9) becomes

$$C_T/k + C_D(k-1)/2 - [C_T/(k-1) + C_D(k-2)/2] < 0 \ , \tag{2.11}$$

which reduces to

$$k(k-1) < 2C_T/C_D . \qquad (2.12)$$

This final relationship allows a very simple iterative computation step to be made starting with k = 1 and repeated until the largest k is found which still satisfies Equation (2.12).

D. DERIVING THE EXPECTED COST FUNCTION FOR MULTIPLE PRODUCTION LINES

McMasters [Ref. 3] derived a formula for the expected delivery and delay costs based on a single demand source over an infinite time horizon. In his derivation, he represented the round trip cost of a delivery by C_T, as was done for the model above. That notation will be retained in this thesis. We will assume that C_T will be incurred each time a delivery takes place. However, if a demand does not occur during an interval of N periods, there will be no cost incurred due to a delivery cancellation.

The delay costs were defined above as being incurred by the NARF as a result of not having a needed part available at the time it is required. Several elements could be considered as part of that cost. These elements include the cost associated with putting the component aside (such as documentation, parts ordered and putting it on the shelf), labor costs due to work stoppage, inventory costs and cost associated with part non-availability to the customer. For

the purpose of this thesis the delay cost will be denoted by C_D.

The expected costs per period for a single production line as derived by McMasters [Ref. 3] is:

$$ECP(N) = [1-(1-p)^N]\left[\frac{-\ell n[1-(1-p)^N]}{(1-p)^N}\right]\left[\frac{C_T}{N} + \frac{pC_D(N-1)}{2}\right] . \quad (2.13)$$

A comparable form can be derived for the multiple production line case. The steps of the derivation follow those of Reference 3.

First the expected total delays for a given production line must be determined. The delays are a function of the number of different configurations that demands can take over the N periods between deliveries. As an example, suppose N = 2, then there are four possible configurations. First, a demand occurs in the first period and none occurs in the second, resulting in a delay of one period. Next, a demand does not occur in the first period, but does occur in the second. This results in a delay of zero. The third configuration for N = 2 is a demand in both the first and second periods. A total delay of one period results from the first period's demand (zero delay for the second period's demand) resulting in a total delay of one period.

The total number of configurations for production 1, when exactly x_ℓ demands occur is

$$n_{x_\ell} = \binom{N_\ell}{x_\ell} . \quad (2.14)$$

Therefore, the total number of configurations which can occur over N periods and have at least one demand is

$$n_\ell = \sum_{x_\ell=1}^{N} n_{x_\ell} = \sum_{x_\ell=1}^{N} \binom{N_\ell}{x_\ell} = 2^{N_\ell}-1 . \qquad (2.15)$$

The expected total delay time for the lth production line associated with the N_ℓ configurations can be determined in the following manner. First consider only those configurations having exactly x_ℓ demands where $x_\ell \geq 1$. The probability of each configuration is:

$$P(x_\ell:N_\ell) = p_\ell^{x_\ell}(1-p_\ell)^{N_\ell-x_\ell} . \qquad (2.16)$$

The number of configurations having a demand in period $1 < i_\ell < N_\ell$ is

$$m_\ell = \binom{N_\ell-1}{x_\ell-1} . \qquad (2.17)$$

It is important to realize that m_ℓ is independent of i_ℓ.

Those demands occurring in period i_ℓ will have to wait until $N_\ell-i_\ell$ periods have passed before the parts are delivered. The total of all delays for these configurations having x_ℓ demands is therefore:

$$TD_\ell(x_\ell,N_\ell) = \sum_{i_\ell=1}^{N_\ell} \binom{N_\ell-1}{x_\ell-1}(N_\ell-i_\ell)$$

$$= \binom{N_\ell-1}{x_\ell-1} \sum_{i_\ell=1}^{N_\ell} (N_\ell-i_\ell)$$

$$= \frac{N_\ell(N_\ell-1)}{2}\binom{N_\ell-1}{x_\ell-1} \cdot \qquad (2.18)$$

From Equations (2.16) and (2.18) it follows that the total expected delays for the lth production line over all values of x_ℓ are:

$$ETD_\ell(N) = \sum_{x_\ell=1}^{N_\ell} TD_\ell(x_\ell,N_\ell)P(x_\ell:N_\ell)$$

$$= \sum_{x_\ell=1}^{N_\ell} \frac{N_\ell(N_\ell-1)}{2}\binom{N_\ell-1}{x_\ell-1} p_\ell^{x_\ell}(1-p_\ell)^{N_\ell-x_\ell}$$

$$= \frac{N_\ell(N_\ell-1)p_\ell}{2} \sum_{x_\ell=1}^{N_\ell} \binom{N_\ell-1}{x_\ell-1} p_\ell^{x_\ell-1}(1-p_\ell)^{N_\ell-x_\ell} \cdot \qquad (2.19)$$

Notice that the summation

$$\sum_{x_\ell=1}^{N_\ell} \binom{N_\ell-1}{x_\ell-1} p_\ell^{x_\ell-1}(1-p_\ell)^{N_\ell-x_\ell} \cdot \qquad (2.20)$$

is the sum of the binomial probabilities over all values of $x_\ell \geq 1$ and its value is 1. Because of this, Equation (2.19) reduces to

$$ETD_\ell(N_\ell) = \frac{N_\ell(N_\ell-1)p_\ell}{2} \quad . \tag{2.21}$$

In developing the expected costs per period over all production lines, consideration must be given to the fact that a delivery will occur at the end of N periods if there has been at least one demand during that time interval. In the case of multiple production lines this also means that at least one line had a demand during N. Using two production lines to begin the development, there are three mutually exclusive events that can occur in N periods which would place a demand on the system. The first is that production line one has at least one requirement and production line two has no requirements. The expected cost per period for the N periods would be

$$ETC(N) = [1-(1-p_1)^N](1-p_2)^N\left[\frac{C_T}{N} + \frac{p_1 C_{D_1}(N-1)}{2}\right] \quad . \tag{2.22}$$

Here $[1-(1-p_1)^N]$ represents the probability that production line one has at least one demand and $(1-p_2)^N$ represents the probability that production line two has no demand in the N periods. Another event is when production line one has no requirements (with probability $(1-p_1)^N$) and production line two has at least one demand (with probability $[1-(1-p_2)^N]$).

The expected cost would be:

$$ETC(N) = [1-(1-p_2)^N](1-p_1)^N[\frac{C_T}{N} + \frac{p_2 C_{D_2}(N-1)}{2}] \cdot \qquad (2.23)$$

Finally production line one has at least one requirement and production line two has at least one requirement. The expected cost would be:

$$ETC(N) = [1-(1-p_1)^N][1-(1-p_2)^N][\frac{C_T}{N} + \frac{p_1 C_{D_1}(N-1)}{2} + \frac{p_2 C_{D_2}(N-1)}{2}] \qquad (2.24)$$

The total expected costs from the two production lines for the N periods is the sum of the costs associated with the three possible events.

$$\begin{aligned} ETC_T(N) &= [1-(1-p_1)^N](1-p_2)^N[\frac{C_T}{N} + \frac{p_1 C_{D_1}(N-1)}{2}] \\ &\quad + [1-(1-p_2)^N](1-p_1)^N[\frac{C_T}{N} + \frac{p_2 C_{D_2}(N-1)}{2}] \\ &\quad + [1-(1-p_1)^N][1-(1-p_2)^N][\frac{C_T}{N} + \frac{p_1 C_{D_1}(N-1)}{2} \\ &\quad + \frac{p_2 C_{D_2}(N-1)}{2}] \\ &= \frac{[1-(1-p_1)^N(1-p_2)^N]C_T}{N} + \frac{[1-(1-p_1)^N]p_1 C_{D_1}(N-1)}{2} \\ &\quad + \frac{[1-(1-p_2)^N]p_2 C_{D_2}(N-1)}{2} \cdot \qquad (2.25) \end{aligned}$$

If three production lines were under consideration the total expected costs for N periods would be as follows:

$$ETC_T(N) = \frac{[1-(1-p_1)^N(1-p_2)^N(1-p_3)^N]C_T}{N} + \frac{[1-(1-p_1)^N]p_1C_{D_1}(N-1)}{2}$$

$$+ \frac{[1-(1-p_2)^N]p_2C_{D_2}(N-1)}{2}$$

$$+ \frac{[1-(1-p_3)^N]p_3C_{D_3}(N-1)}{2} \quad . \qquad (2.26)$$

For L production lines, the total expected delay cost function for the first N periods would be:

$$ETC_T(N) = \frac{[1-(1-p_1)^N(1-p_2)^N\cdots(1-p_L)^N]C_T}{N}$$

$$+ \frac{[1-(1-p_1)^N]p_1C_{D_1}(N-1)}{2}$$

$$+ \frac{[1-(1-p_2)^N]p_2C_{D_2}(N-1)}{2}$$

$$+ \ldots + \frac{[1-(1-p_L)^N]p_LC_{D_L}(N-1)}{2} \quad . \qquad (2.27)$$

The generalized form for a finite number of production lines, L, for N periods is

$$ETC_T(N) = [1 - \prod_{\ell=1}^{L}(1-p_\ell)^N]\frac{C_T}{N} + \sum_{\ell=1}^{L}\frac{[1-(1-p_\ell)^N]p_\ell C_{D_\ell}(N-1)}{2} \quad . \qquad (2.28)$$

Now there are actually four mutually exclusive events that can take place if there are two production lines under consideration. The fourth event is that no demands will occur in either production lines one or two. The probability of this event is $(1-p_1)^N(1-p_2)^N$. Because of this event, a renewal theory approach which considers a sequence of many intervals of N periods is appropriate. A renewal occurs if a delivery occurs. The first three events of the two production line model correspond to a renewal in the first N periods whereas the fourth corresponds to renewals in later N period intervals. In this latter case consideration must be given to what the expected total costs will be in the second set of N periods. For the two production lines, the form of the expected total costs over the second N periods depends on four mutually exclusive events that can take place. The first is that neither production line one nor two places a demand in the first N periods but production line one has a demand during the second N periods. The expected costs per period are:

$$ETC(2N) = (1-p_1)^N(1-p_2)^N[1-(1-p_1)^N](1-p_2)^N[\frac{C_T}{2N} + \frac{p_1 C_{D_1}(N-1)}{4}]. \quad (2.29)$$

The second event is the same as the first except that line two has a demand and line one does not. The expected costs per period are:

$$ETC(2N) = (1-p_1)^N(1-p_2)^N[1-(1-p_2)^N](1-p_1)^N \cdot [\frac{C_T}{2N} + \frac{p_2 C_{D_2}^{(N-1)}}{4}] . \quad (2.30)$$

The third event is when both lines one and two only have demands during the second N periods. The expected delay costs per period are

$$ETC(2N) = (1-p_1)^N(1-p_2)^N[1-(1-p_1)^N][1-(1-p_2)^N] \cdot [\frac{C_T}{2N} + \frac{p_1 C_{D_1}^{(N-1)}}{4} + \frac{p_2 C_{D_2}^{(N-1)}}{4}] \cdot \quad (2.31)$$

The fourth event is that no demands occur during the second N periods. Expected costs per period when a renewal spans 2N periods is the sum of the above three events.

$$ETC_T(2N) = (1-p_1)^N(1-p_2)^N[1-(1-p_1)^N](1-p_2)^N[\frac{C_T}{2N} + \frac{p_1 C_{D_1}^{(N-1)}}{4}]$$
$$+ (1-p_1)^N(1-p_2)^N[1-(1-p_2)^N](1-p_1)^N[\frac{C_T}{2N} + \frac{p_2 C_{D_2}^{(N-1)}}{4}]$$
$$+ (1-p_1)^N(1-p_2)^N[1-(1-p_1)^N][1-(1-p_2)^N] \cdot [\frac{C_T}{2N} + \frac{p_1 C_{D_1}^{(N-1)}}{4} + \frac{p_2 C_{D_2}^{(N-1)}}{4}]$$

This simplifies into Equation (2.32):

$$ETC_T(2N) = \frac{(1-p_1)^N(1-p_2)^N}{2}\{\frac{C_T[1-(1-p_1)^N(1-p_2)^N]}{N} + \frac{p_1 C_{D_1}(N-1)[1-(1-p_1)^N]}{2} + \frac{p_2 C_{D_2}(N-1)[1-(1-p_2)^N]}{2}\} \cdot \quad (2.32)$$

If we apply the generalized form for more than two production lines, the expected total costs per period associated with the second set of N periods is

$$ETC_T(2N) = \prod_{\ell=1}^{L} \frac{(1-p_\ell)^N}{2}\{\frac{C_T[1-\prod_{\ell=1}^{L}(1-p_\ell)^N]}{N} + \sum_{\ell=1}^{L} \frac{p_\ell C_{D_\ell}(N-1)[1-(1-p_\ell)^N]}{2}\} \cdot \quad (2.33)$$

If no demands occur during the first and second sets of N periods, the third set of N periods must be considered. Assuming there has been no demand in the first and second intervals, then the probability associated with this occurrence would be $(1-p_1)^{2N}(1-p_2)^{2N}$ in the two production lines case. The expected total costs per period due to delay when 3N periods occur before a renewal is

$$ETC_T(3N) = \frac{(1-p_1)^{2N}(1-p_2)^{2N}}{3}\left\{\frac{C_T[1-(1-p_1)^N(1-p_2)^N]}{N} + \frac{p_1 C_{D_1}(N-1)[1-(1-p_1)^N]}{2} + \frac{p_2 C_{D_2}(N-1)[1-(1-p_2)^N]}{2}\right\} . \quad (2.34)$$

The generalized form for more than two production lines for the third interval becomes

$$ETC_T(3N) = \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{2N}}{3}\left\{\frac{C_T[1-\prod_{\ell=1}^{L}(1-p_\ell)^N]}{N} + \sum_{\ell=1}^{L}\frac{p_\ell C_{D_\ell}(N-1)[1-(1-p_\ell)^N]}{2}\right\} . \quad (2.35)$$

Generalizing to k intervals of N periods, the total expected costs per period for L production lines is:

$$ETC_T(kN) = \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{(k-1)N}}{k}\left\{\frac{C_T[1-\prod_{\ell=1}^{L}(1-p_\ell)^N]}{N} + \sum_{\ell=1}^{L}\frac{p_\ell C_{D_\ell}(N-1)[1-(1-p_\ell)^N]}{2}\right\} \quad (2.36)$$

Finally, the total expected costs per period over all future periods can be obtained by summing over all possible k values:

$$ETC_T(N) = \sum_{k=1}^{\infty} \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{(k-1)N}}{k} \left\{ \frac{C_T\left[1-\prod_{\ell=1}^{L}(1-p_\ell)^N\right]}{N} + \sum_{\ell=1}^{L} \frac{p_\ell C_{D_\ell}(N-1)\left[1-(1-p_\ell)^N\right]}{2} \right\} . \qquad (2.37)$$

The summation term in Equation (2.37) can be rewritten as follows:

$$\sum_{k=1}^{\infty} \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{(k-1)N}}{k} = \frac{1}{\prod_{\ell=1}^{L}(1-p_\ell)^N} \sum_{k=1}^{\infty} \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{kN}}{k} ,$$

and since

$$\frac{1}{a} \sum_{k=1}^{\infty} \frac{a^k}{k} = \frac{1}{a}[-\ln(1-a)] ,$$

$$\sum_{k=1}^{\infty} \frac{\prod_{\ell=1}^{L}(1-p_\ell)^{(k-1)N}}{k} = \frac{-\ln\left[1-\prod_{\ell=1}^{L}(1-p_\ell)^N\right]}{\prod_{\ell=1}^{L}(1-p_\ell)^N}$$

Therefore Equation (2.37) reduces to

$$ETC_T(N) = \left\{\frac{-\ell n\left[1 - \prod_{\ell=1}^{L}(1-p_\ell)^N\right]}{\prod_{\ell=1}^{L}(1-p_\ell)^N}\right\}\left\{\frac{\left[1-\prod_{\ell=1}^{L}(1-p_\ell)^N\right]C_T}{N}\right\}$$

$$+ \sum_{\ell=1}^{L} \frac{p_\ell C_{D_\ell}(N-1)\left[1-(1-p_\ell)^N\right]}{2}\Bigg\} . \qquad (2.38)$$

E. DETERMINATION OF OPTIMAL N

Since N can take on only integer values, it is necessary to use finite differences to determine the optimal value for N. Optimal N is that value of N which satisfies the following:

$$ETC_T(N-1) > ETC_T(N) \leq ETC_T(N+1) . \qquad (2.39)$$

This is the same as saying that optimal N is the largest value of N such that

$$\Delta ETC_T(N) = ETC_T(N) - ETC_T(N-1) < 0 . \qquad (2.40)$$

However, if one attempts to evaluate the form of $\Delta ETC(N)$, it becomes quickly apparent that the result is more complex than $ETC_T(N)$ itself. Therefore, the determination of optimal N in the next chapter consists of evaluating $ETC_T(N)$ for increasing N as long as inequality (2.40) is satisfied.

F. COST FUNCTION IF PROBABILITY OF DEMAND EQUALS ONE

For purposes of comparison in the next chapter the special case of Equation (2.38) when $p_1 = p_2 = \ldots = p_L = 1$ will be useful. In that case, Equation (2.38) reduces to:

$$ETC_T(N) = \frac{C_T}{N} + \sum_{\ell=1}^{L} \frac{C_{D_\ell}(N-1)}{2} \qquad (2.41)$$

which is a simple extension of Equation (2.7) and a delivery will occur at the end of the first N periods.

III. OPTIMIZATION ANALYSIS

A. RELEVANT VARIABLES IN DETERMINING AN OPTIMAL N

Determination of an optimal N is centered around the influence of the cost of transportation from the supply point to the NARF and return, the cost due to delay, the probability of demand and the number of production lines. The intent of the optimization analysis will be to show how N varies with respect to p_ℓ, C_{D_ℓ}, and l. The transportation cost C_T will be fixed at \$100 throughout the analysis since it is independent of the number of production lines.

B. GENERAL BEHAVIOR OF THE COST CURVES

Figures 3.1 through 3.5 are graphical representations of the expected total costs per period and its two component parts, transportation costs and delay costs per period. Because the domain of the cost function is integer valued, each point on a graph represents the cost at that N value. The points have been connected for better visualization of a cost function's behavior. Figures 3.1 through 3.3 are for a single production line and show the effect of increasing C_D. As the value of C_D is increased, the value for the optimal N decreases. In all cases the value for the C_D curve starts at zero when N = 1 and the slope of the curve, as the value of C_D increases, becomes steeper. The $ETC_T(N)$ curve approaches the C_D curve asymptotically.

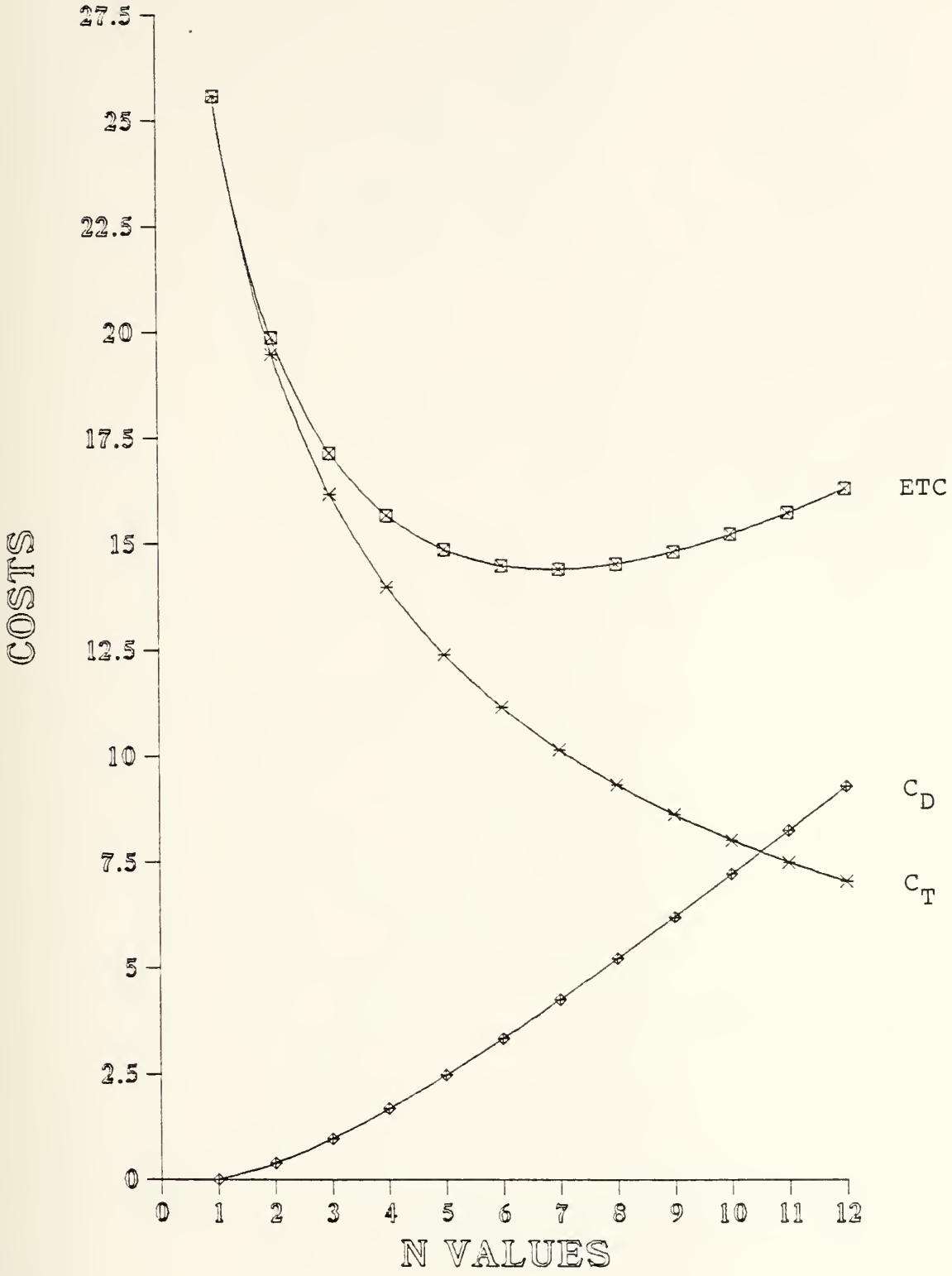

Figure 3.1 Expected Cost Curves as a Function of N for a Single Production Line When C_D = \$10 and p = 0.1

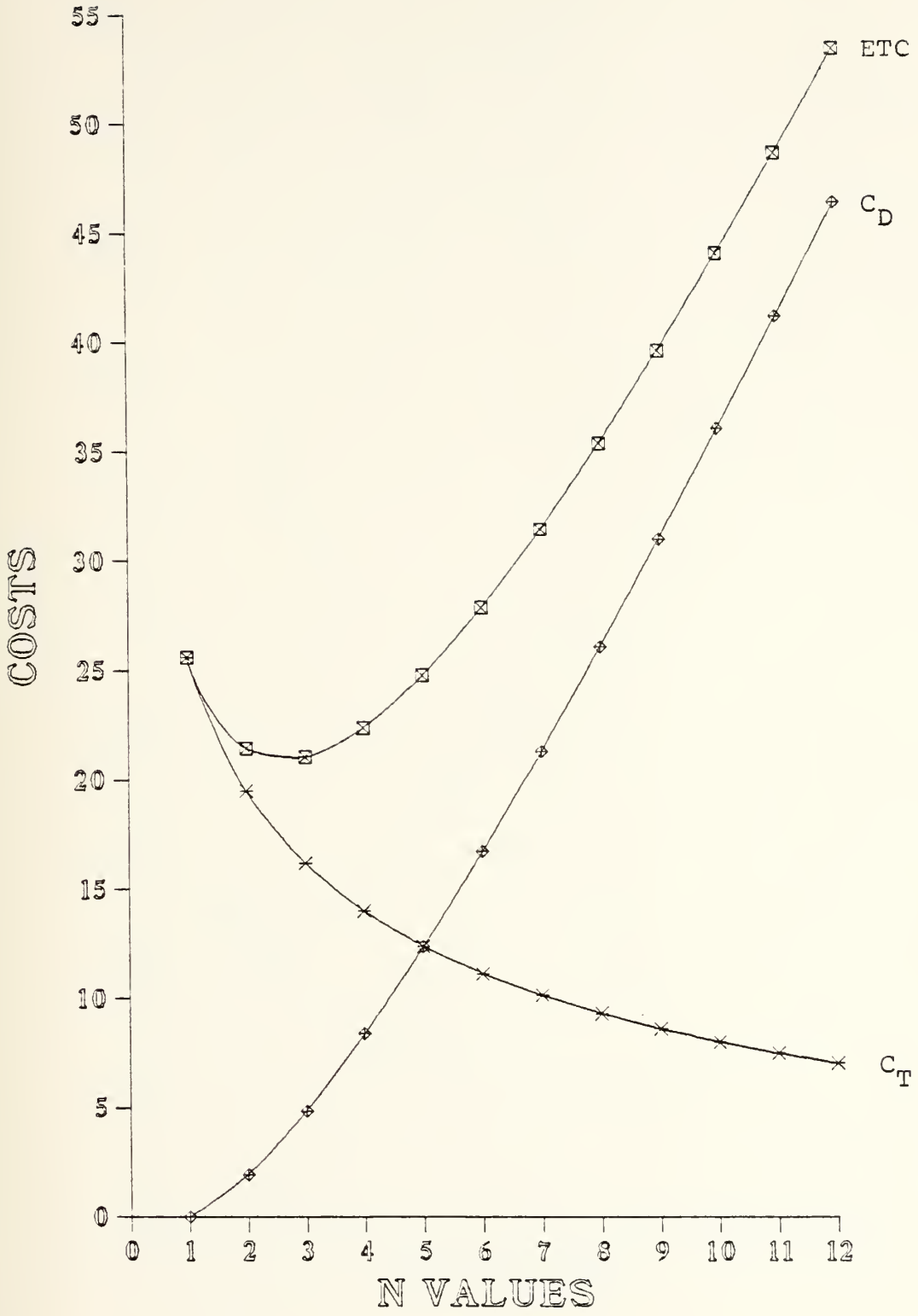

Figure 3.2 Expected Cost Curves as a Function of N for a Single Production Line When C_D = \$50 and p = 0.1

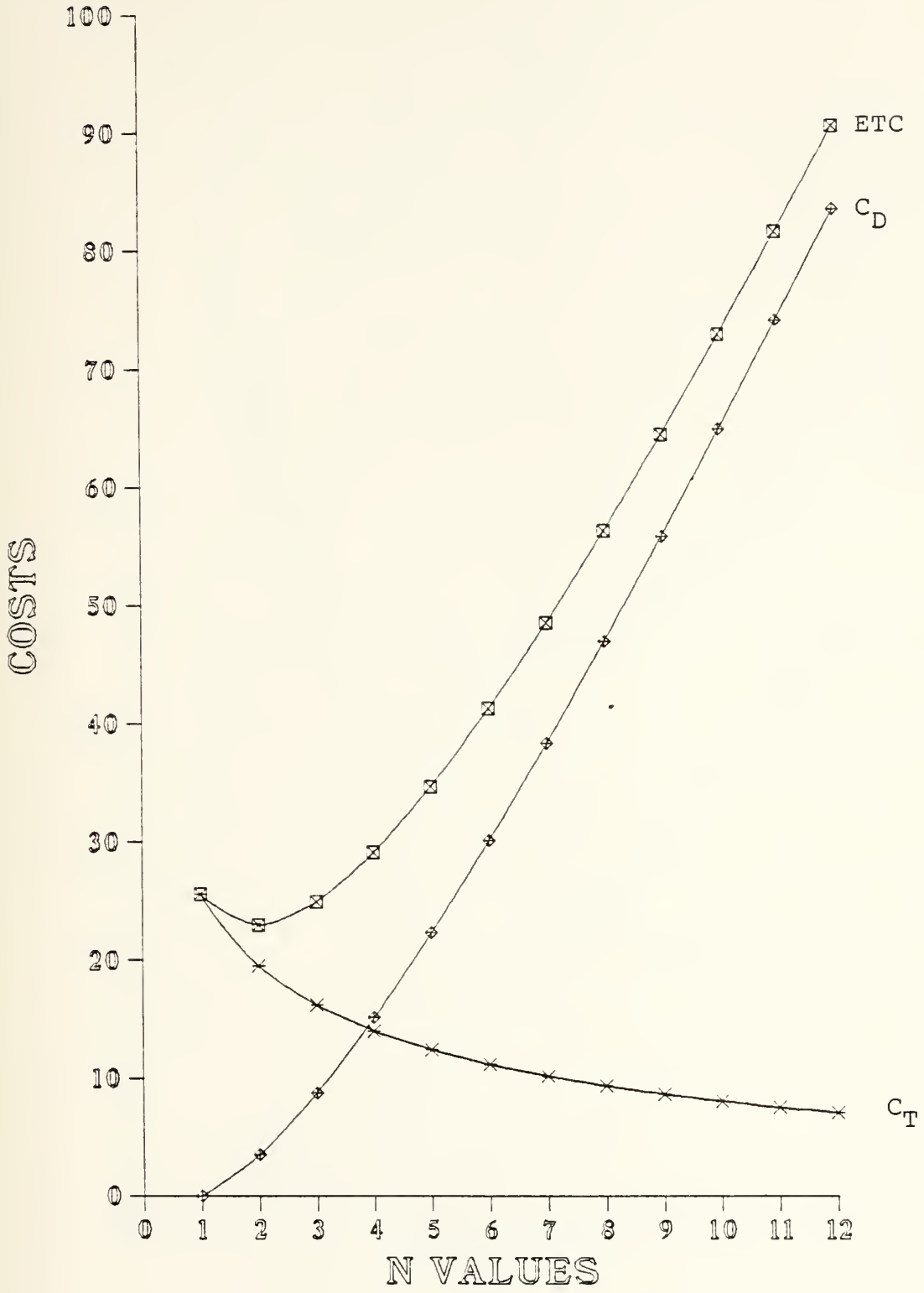

Figure 3.3 Expected Cost Curves as a Function of N for a Single Production Line When C_D = \$90 and p = 0.1

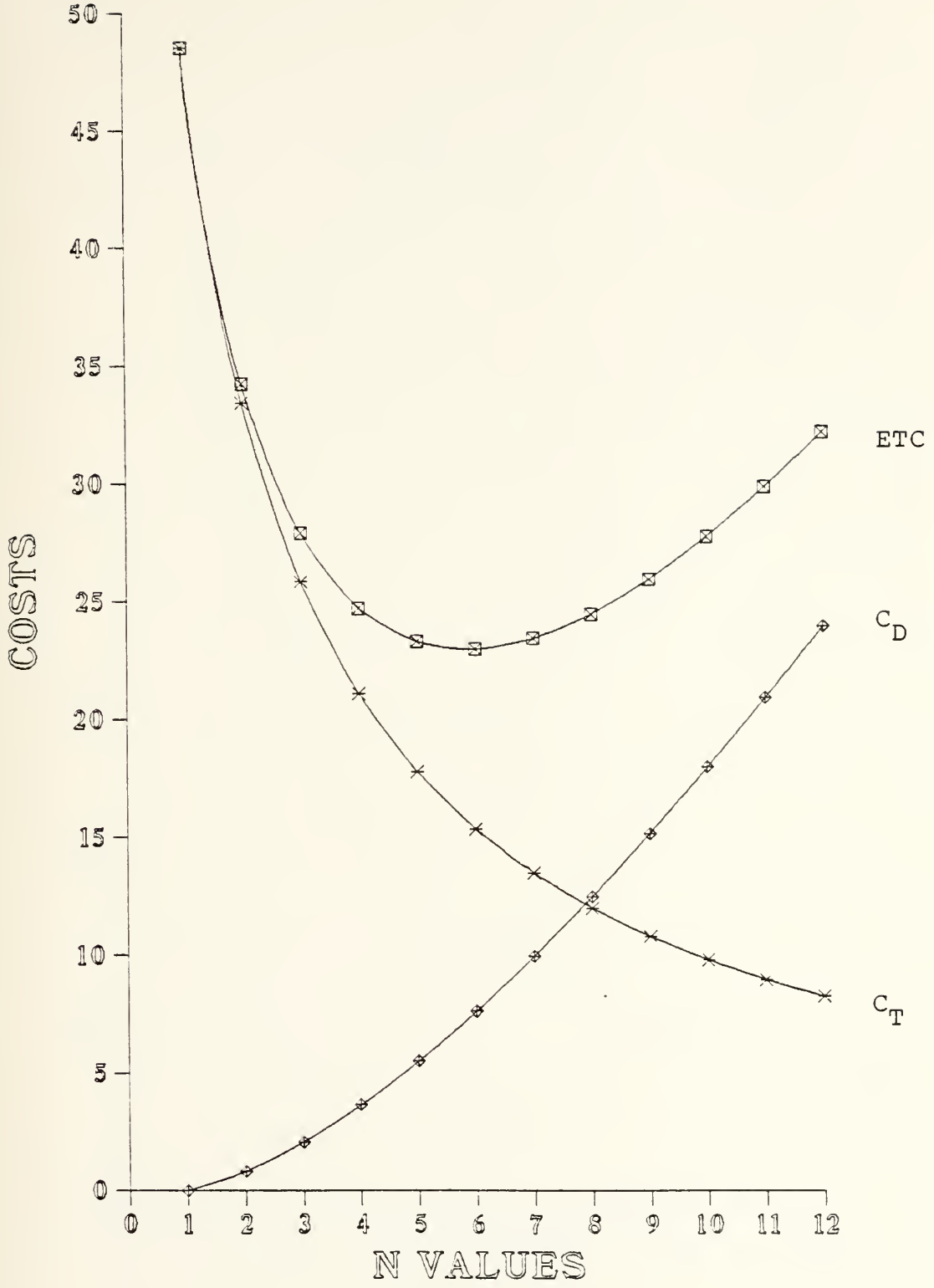

Figure 3.4 Expected Cost Curves as a Function of N for Three Production Lines When $C_{D_1} = C_{D_2} = C_{D_3} = \$10$ and $p_1 = p_2 = p_3 = 0.1$

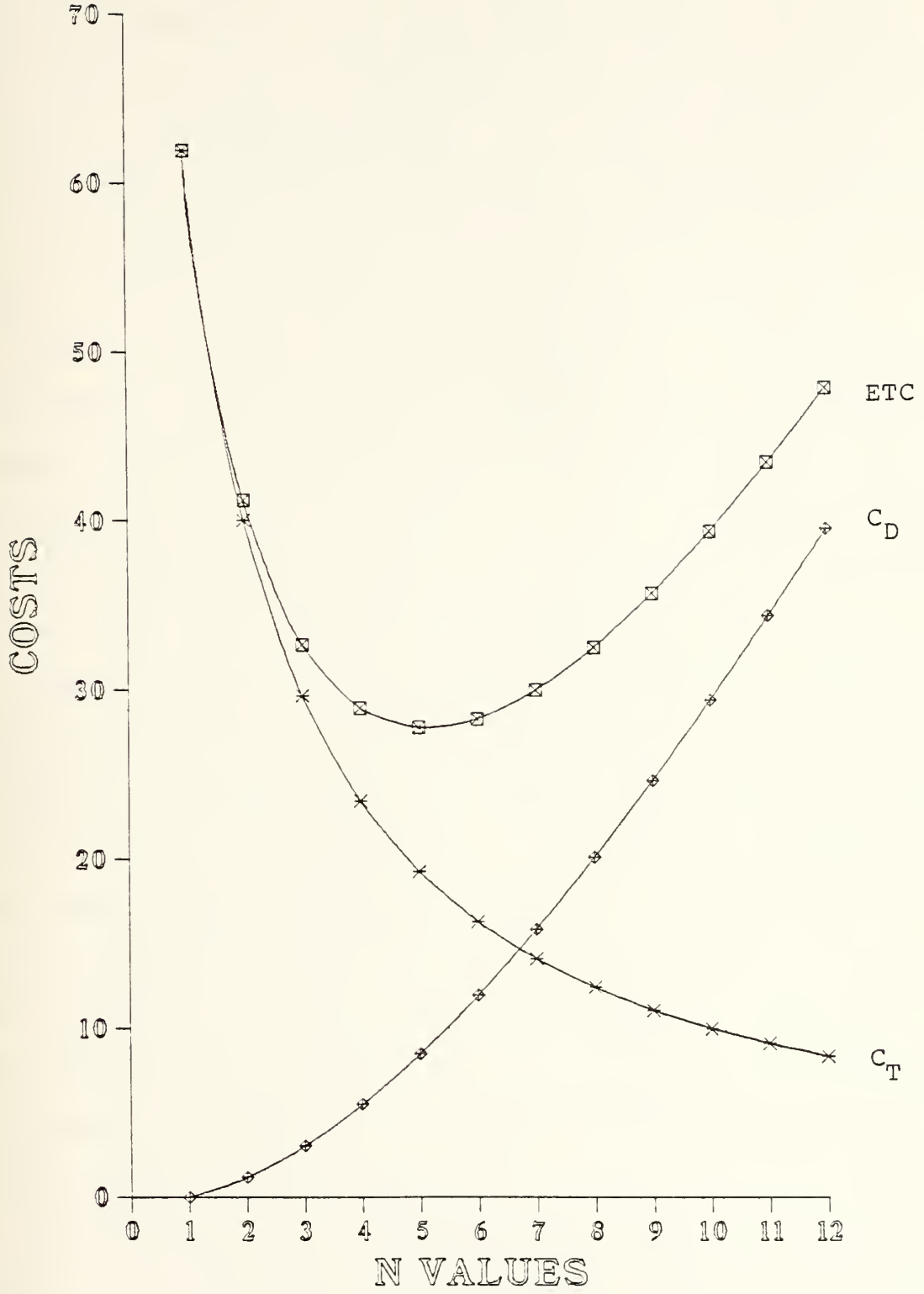

Figure 3.5 Expected Cost Curves as a Function of N for Five Production Lines When $C_{D_1} = \ldots = C_{D_5} = \$10$ and $p_1 = \ldots = p_5 = 0.1$

Figures 3.1, 3.4 and 3.5 illustrate the impact from the number of production lines. Both the transportation and delay cost components are affected. In these figures the p_ℓ and C_{D_ℓ} values are set equal; $p_\ell = 0.1$ and $C_{D_\ell} = \$10$. As the number of lines increases the C_T cost term increases. It is most significant for N = 1. The influence from the number of lines then decreases rapidly as N increases.

The C_D cost term is always zero when N = 1 regardless of the number of production lines. However, as N increases it increases at an increasing rate. When there are more than one production lines, the rate of increase is faster because all of the lines experience delays. The additive effect is however less than linear with the number of lines.

Figures 3.6 through 3.8 present the effects of various parameter changes on ETC_T for a two production lines case. In these figures C_{D_1} and p_1 are fixed at \$10 and 0.1, respectively. Figure 3.6 then sets N = 2 and shows the effects that changing C_{D_2} and p_2 have on $ETC_T(2)$. Figures 3.7 and 3.8 show similar results for N = 3 and N = 5, respectively. For a given C_{D_2} value the total costs appear to increase with increasing N. However it is not true for all p_2 values as Figures 3.9 and 3.10 show. These figures have C_{D_2} fixed at \$10 and \$20, respectively.

Figures such as 3.9 and 3.10 can be used to determine optimal N as a function of p_2. Optimal N corresponds to the minimum ETC_T curve for any given p_2 value. For example,

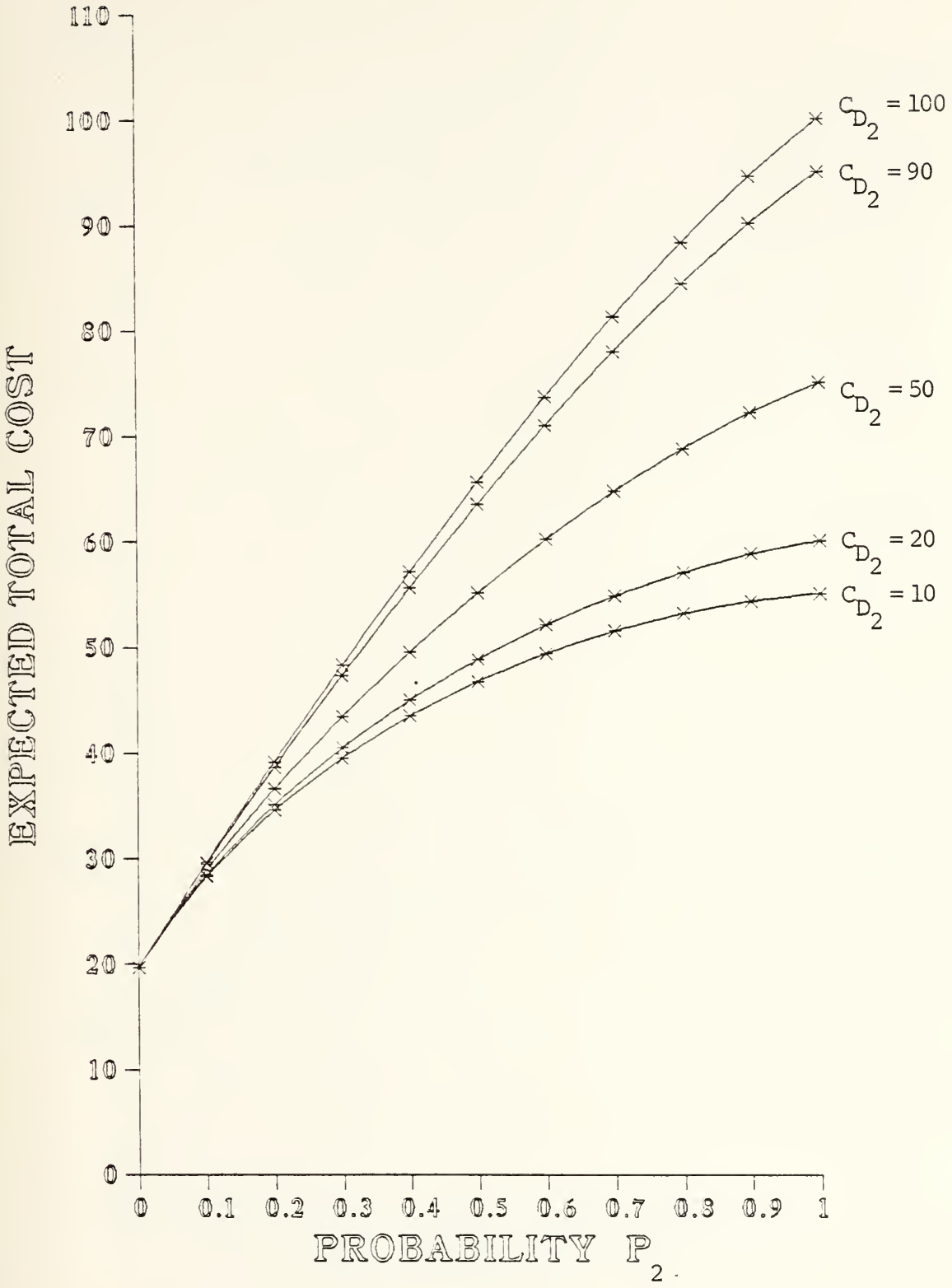

Figure 3.6 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When C_{D_2} is Varied and C_{D_1} = \$10, p_1 = 0.1 and N = 2

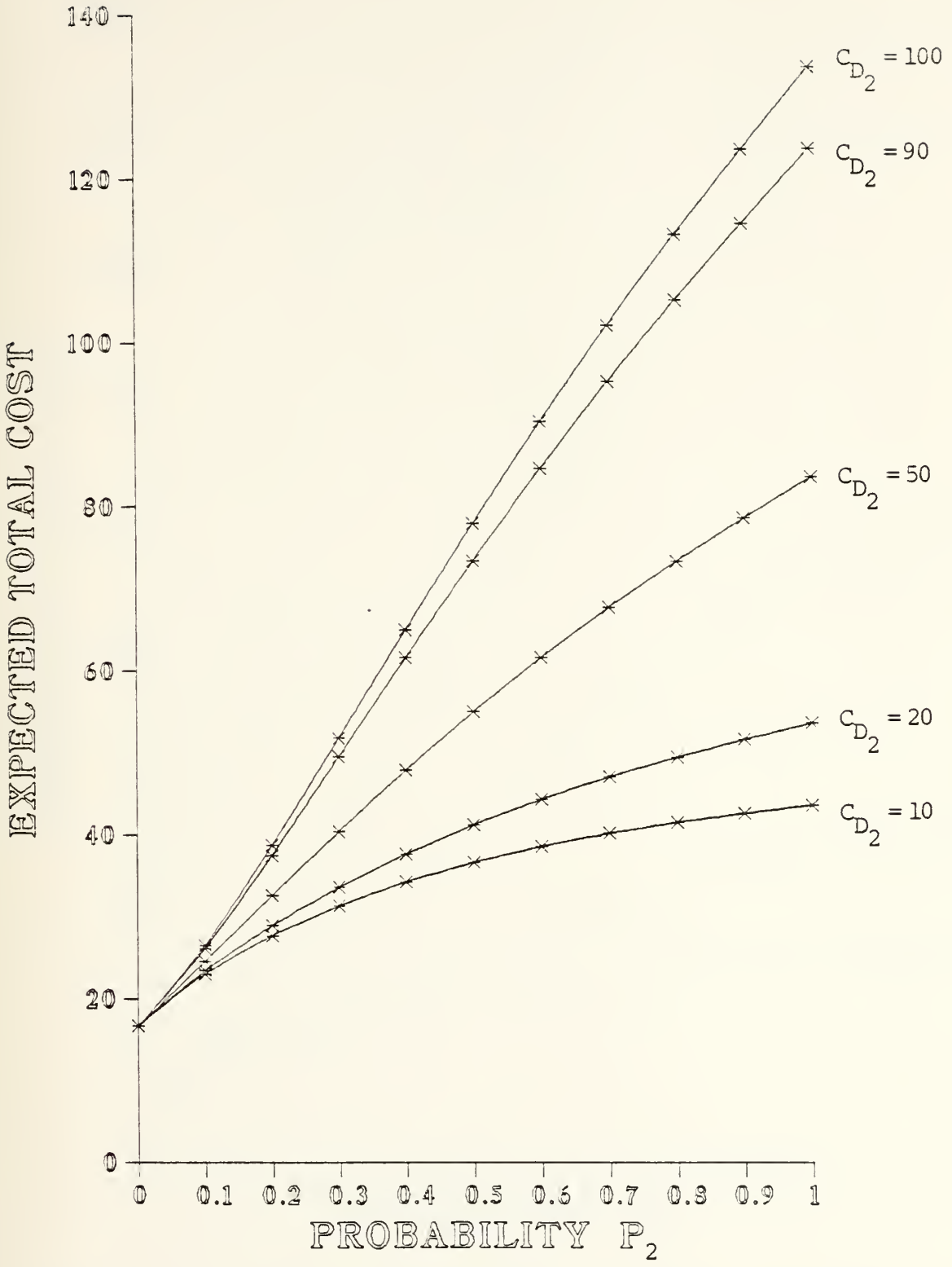

Figure 3.7 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When C_{D_2} is Varied and C_{D_1} = \$10, p_1 = 0.1 and N = 3

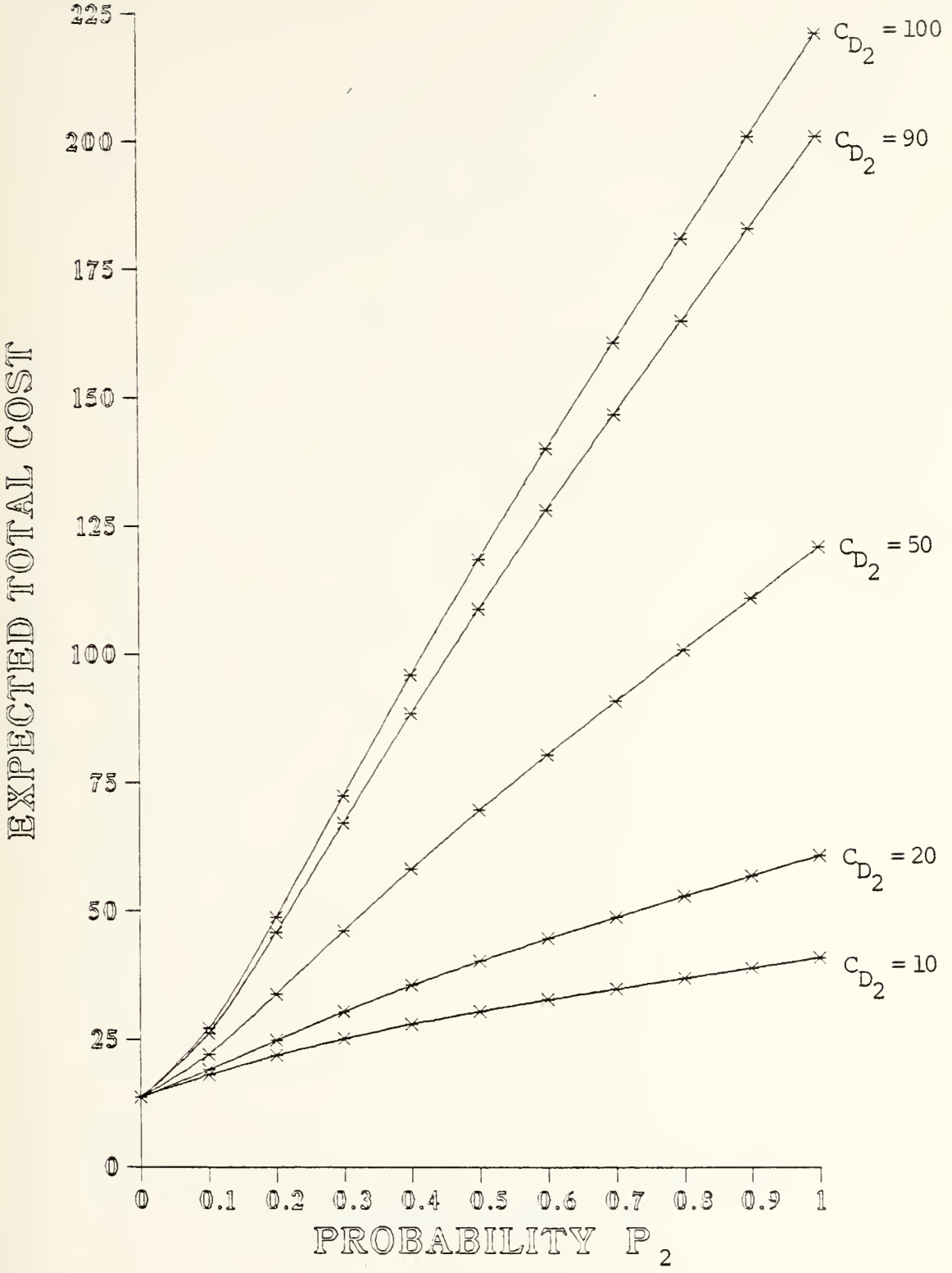

Figure 3.8 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When C_{D_2} is Varied and C_{D_1} = \$10, p_1 = 0.1 and N = 5

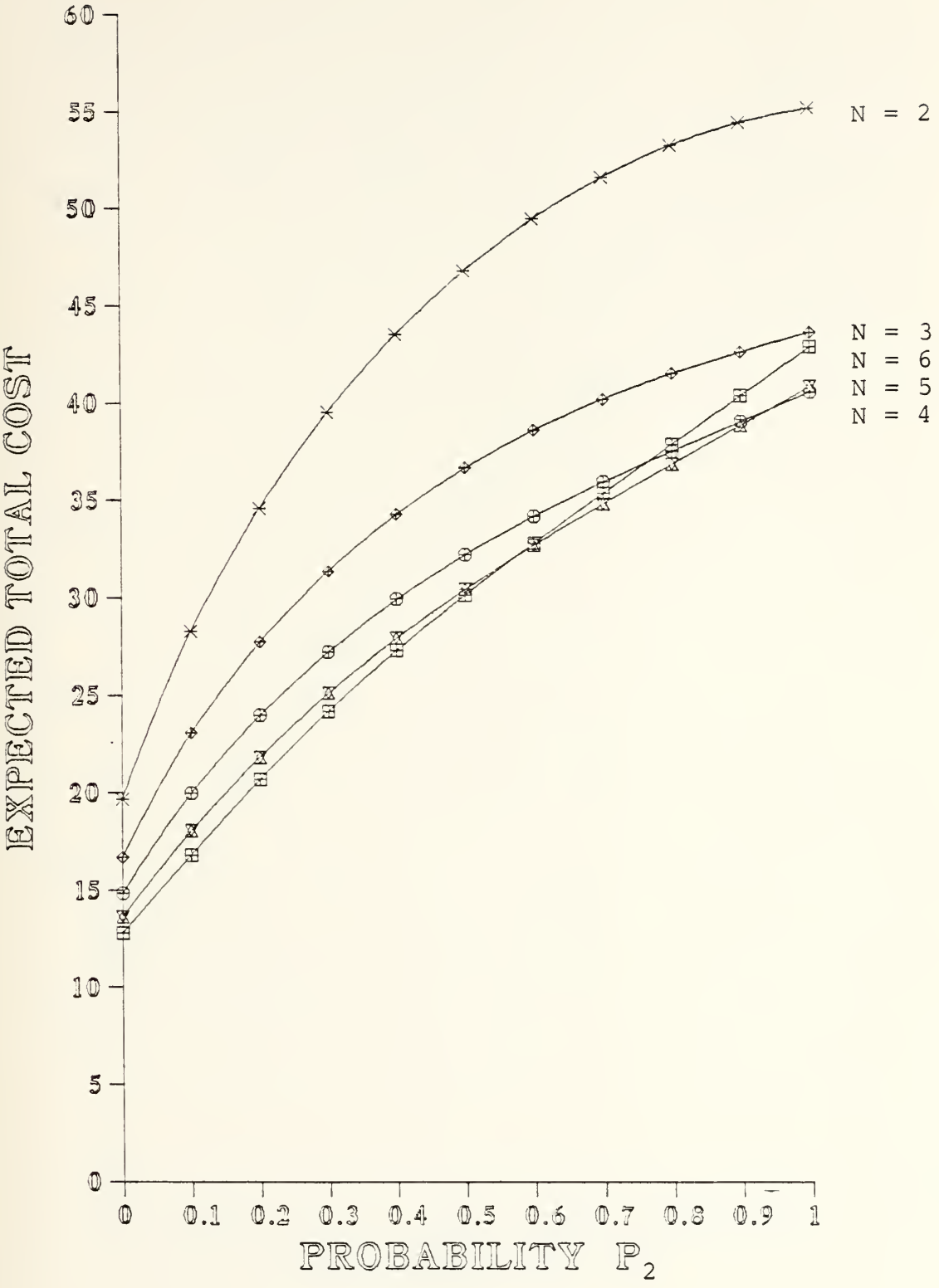

Figure 3.9 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When N is Varied, $C_{D_1} = C_{D_2} = \$10$, and $p_1 = 0.1$

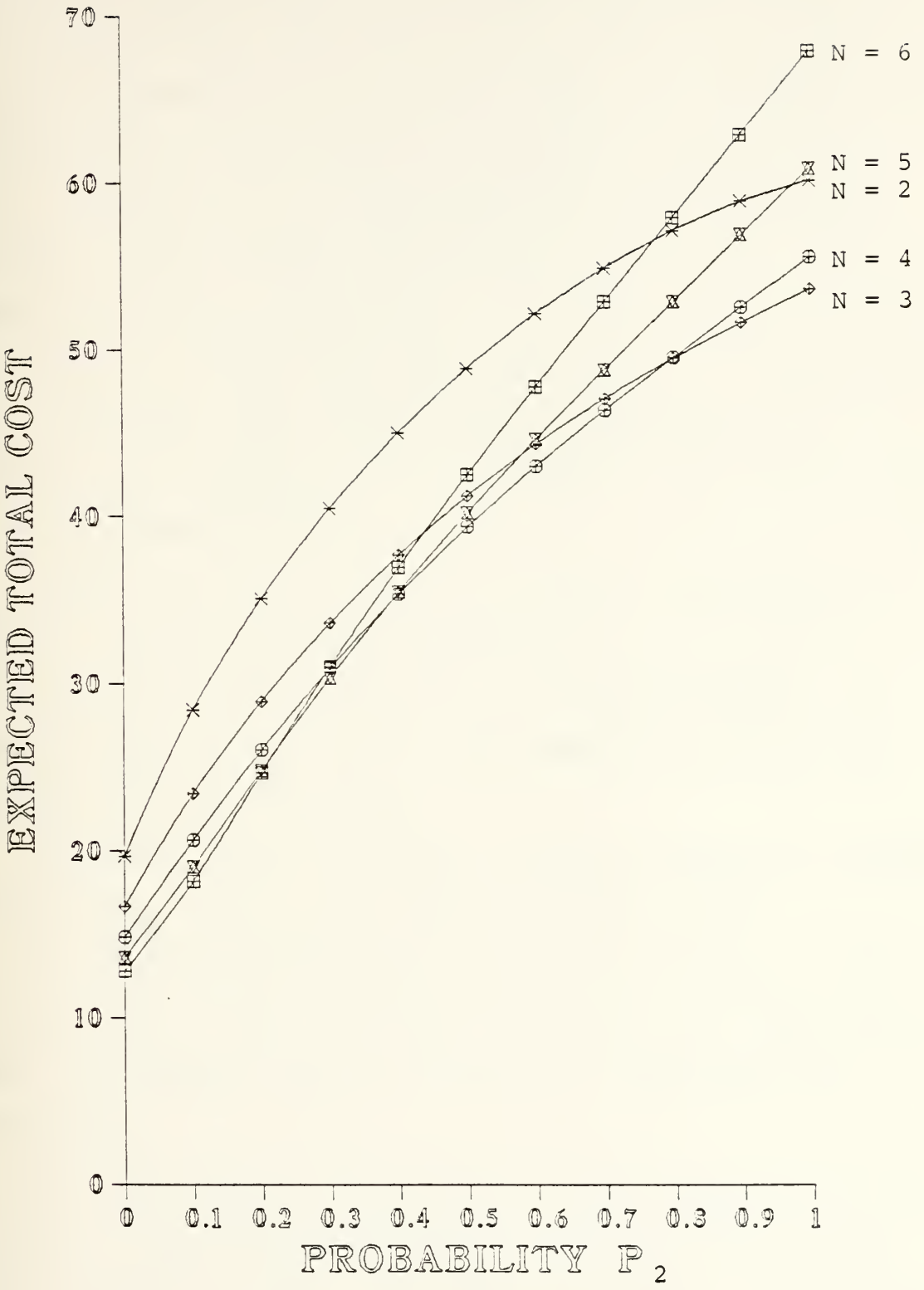

Figure 3.10 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When N is Varied, C_{D_1} = \$10, C_{D_2} = \$20 and p_1 = .1

Figure 3.9 shows that N = 2 is never opimal for the values of the parameters used in our study, that N = 4 is optimal for p_2 from approximately 0.93 to 1.0, that N = 5 is optimal for p_2 from 0.58 to 0.93 and N = 6 is optimal from p_2 = 0 to 0.58 (actually N = 6 is not optimal over the whole range because larger N values yield lower cost curves). Figure 3.11 shows the plot of optimal N as a function of p_2 for the data from Figure 3.9 plus additional curves for N = 7 through 12. As is expected, the optimal value of N decreases with increasing p_2, the probability of a demand in any period from a second production line.

Figure 3.12 shows the plot of optimal N provided by Figure 3.10 plus additional curves for N = 7, 8, and 9. Comparison of Figures 3.11 and 3.12 shows the impact of changing C_{D_2} from \$10 to \$20. As expected, optimal N decreases with increasing C_{D_2}.

Figures 3.13 and 3.14 show a dramatic decrease in optimal N because C_{D_1} and C_{D_2} have been increased to \$50.

Figure 3.15 shows optimal N when p_1 = 0.5, C_{D_1} = \$10 and C_{D_2} = \$20. This figure when compared with Figure 3.14 shows the expected result that optimal N decreases as p increases.

Finally, Figures 3.16 and 3.17 present results for three production lines when $p_1 = p_2 = 0.1$ and C_{D_ℓ} = \$10 for all three production lines. The effect of adding a line can be seen by comparing Figure 3.17 with 3.11. The result is that

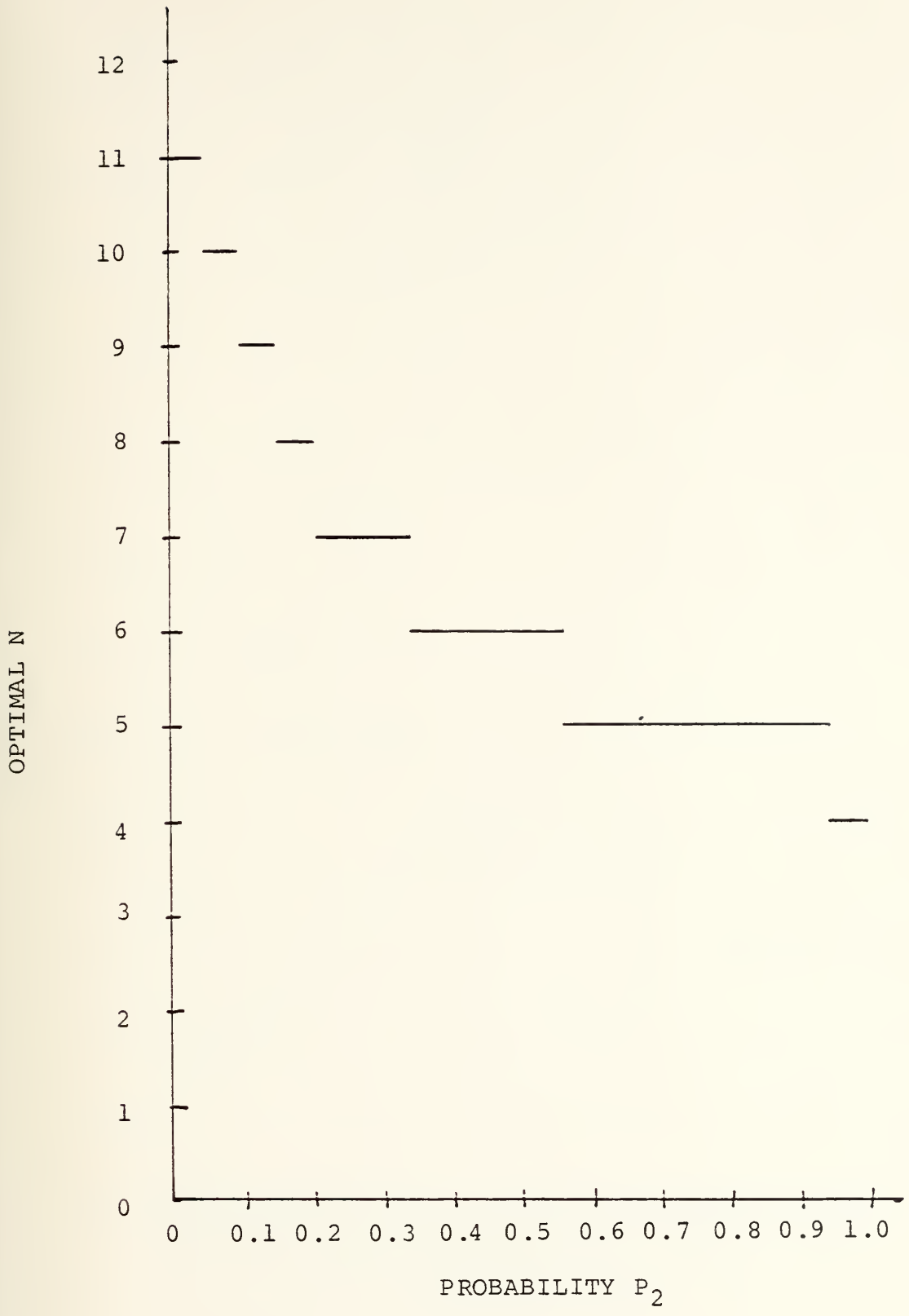

Figure 3.11 Optimal N for Two Production Lines When $p_1 = 1$, $C_{D_1} = C_{D_2} = 10$

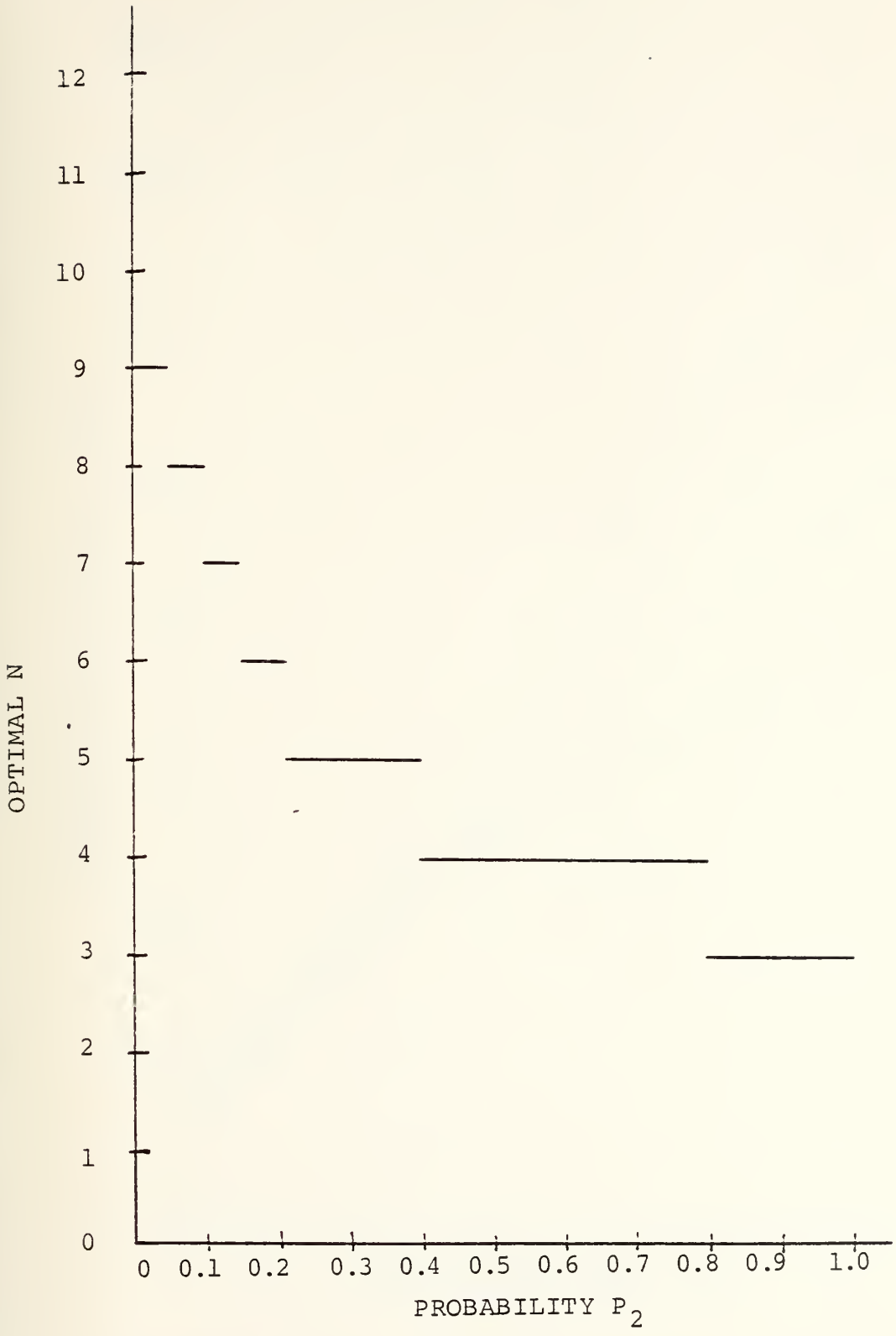

Figure 3.12 Optimal N for Two Production Lines When C_{D_1} = \$10, C_{D_2} = \$20 and p_1 = 0.1

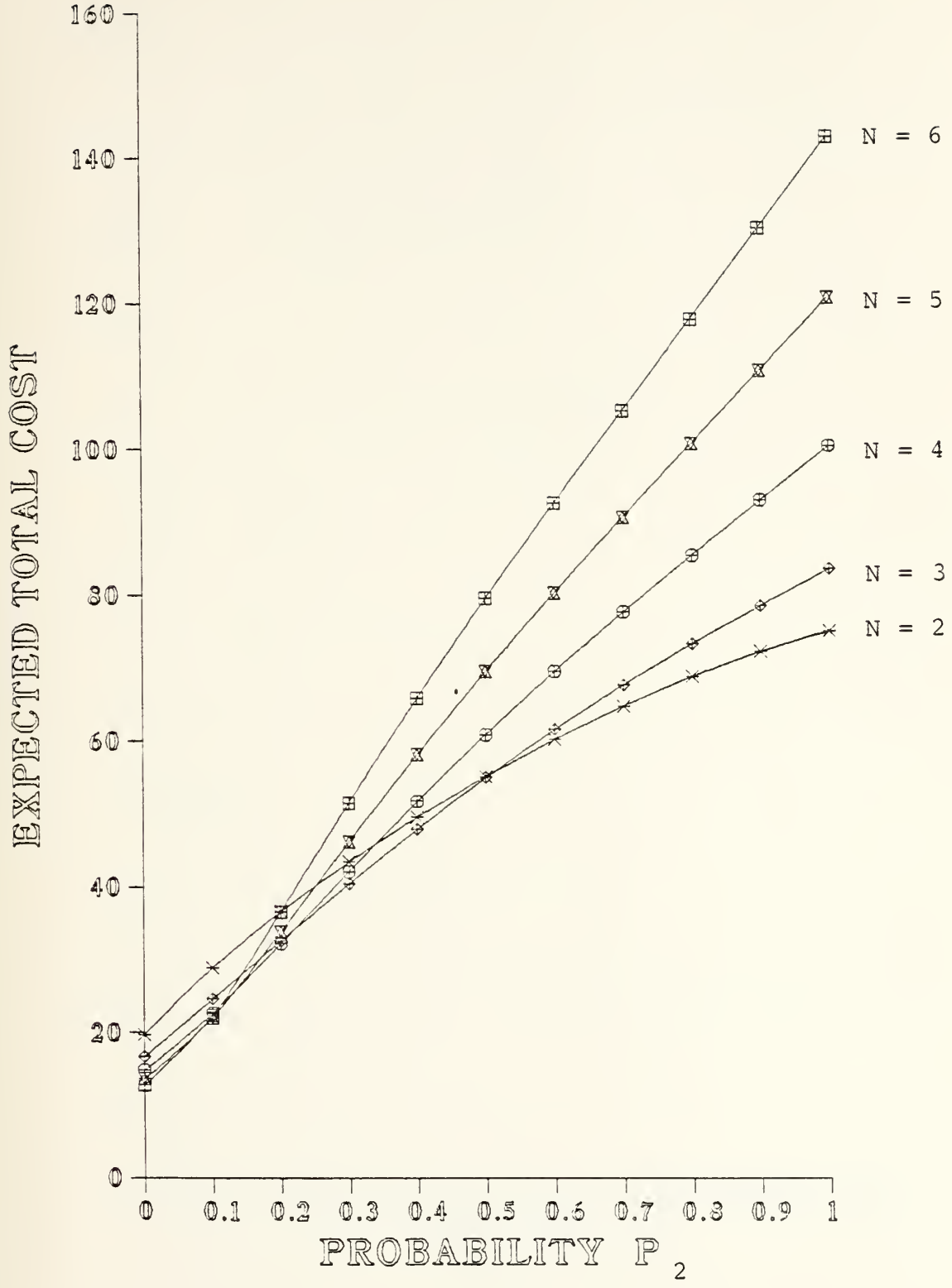

Figure 3.13 Expected Total Cost Curves as a Function of p_2 for Two Production Lines When N is Varied, $C_{D_1} = C_{D_2} = \$50$ and $p_1 = 0.1$

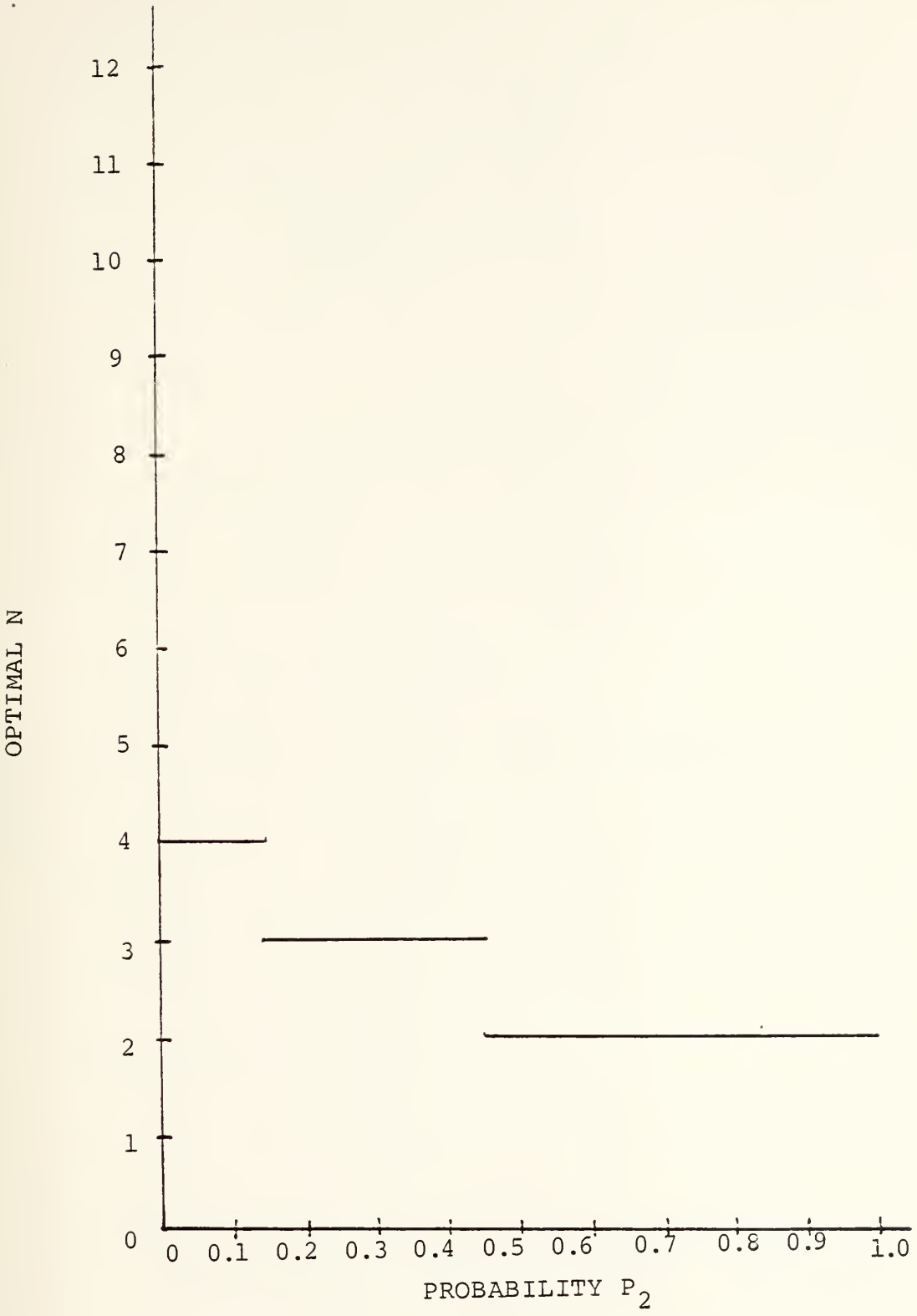

Figure 3.14 Optimal N for Two Production Lines When $C_{D_1} = C_{D_2} = \$50$ and $p_1 = 0.1$

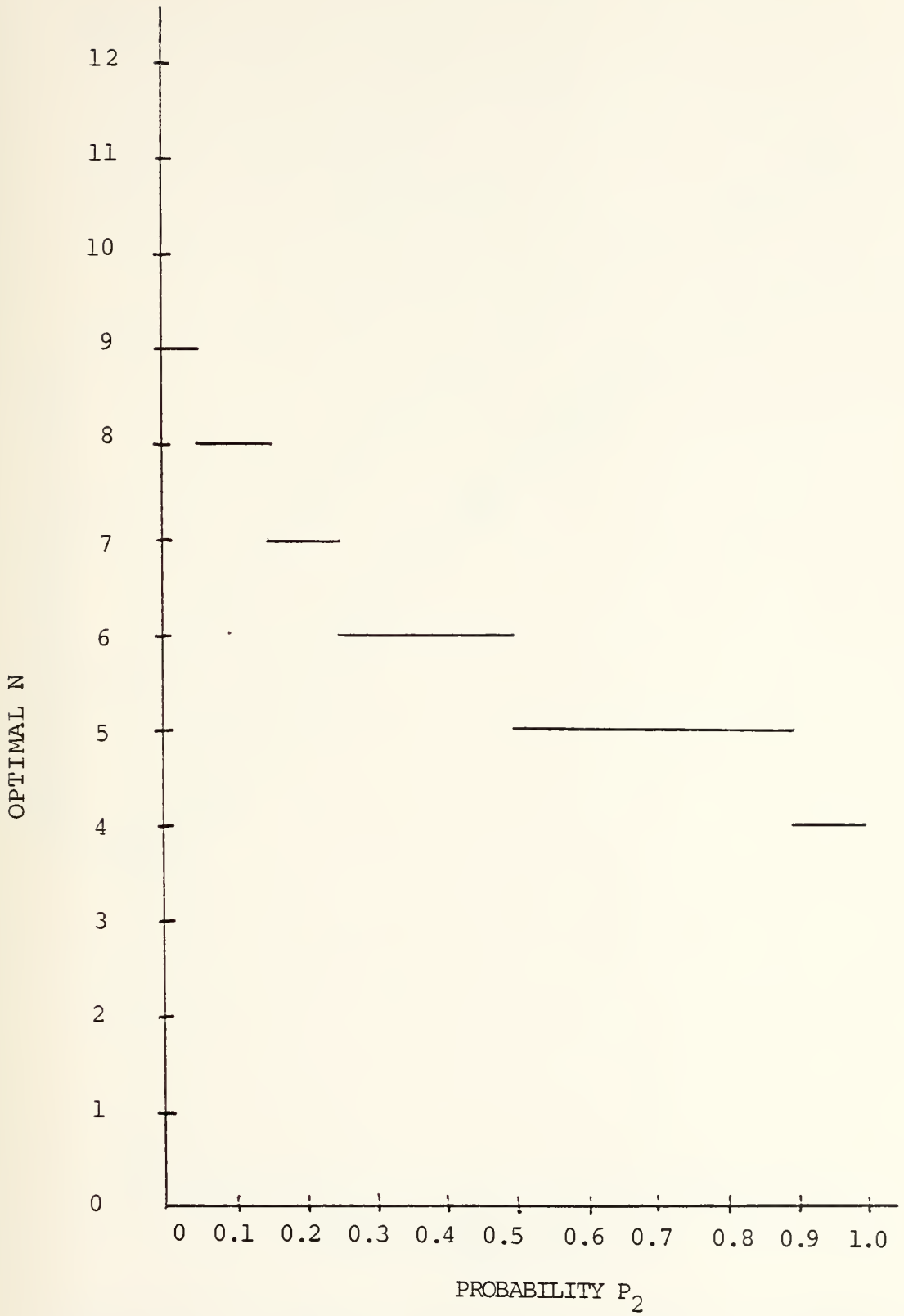

Figure 3.15 Optimal N for Two Production Lines When C_{D_1} = \$10, C_{D_2} = \$20 and p_1 = 0.5

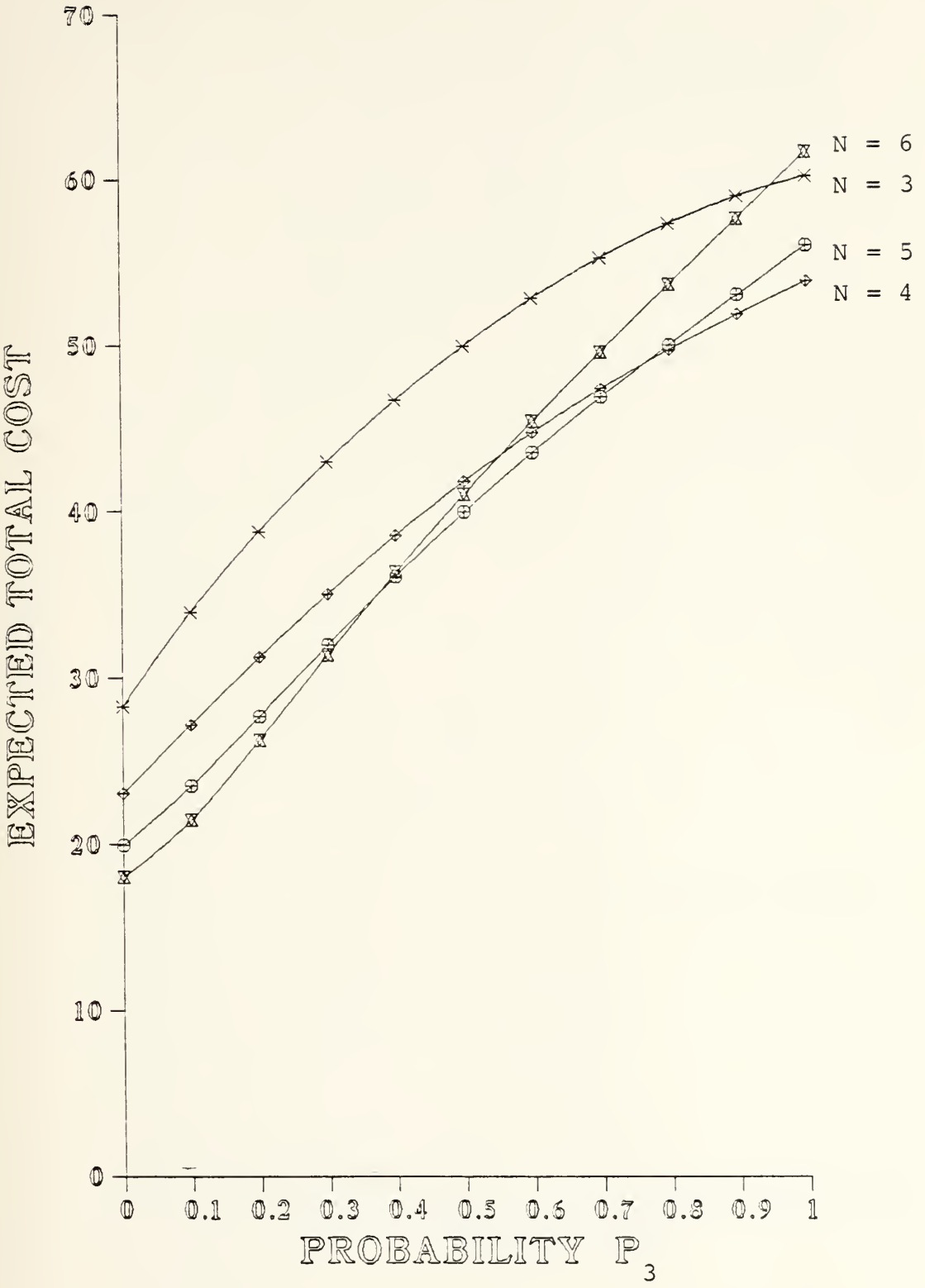

Figure 3.16 Expected Total Cost Curves as a Function of p_2 for Three Production Lines When N is Varied, $C_{D_1} = C_{D_2} = C_{D_3} = \$10$ and $p_1 = p_2 = 0.1$

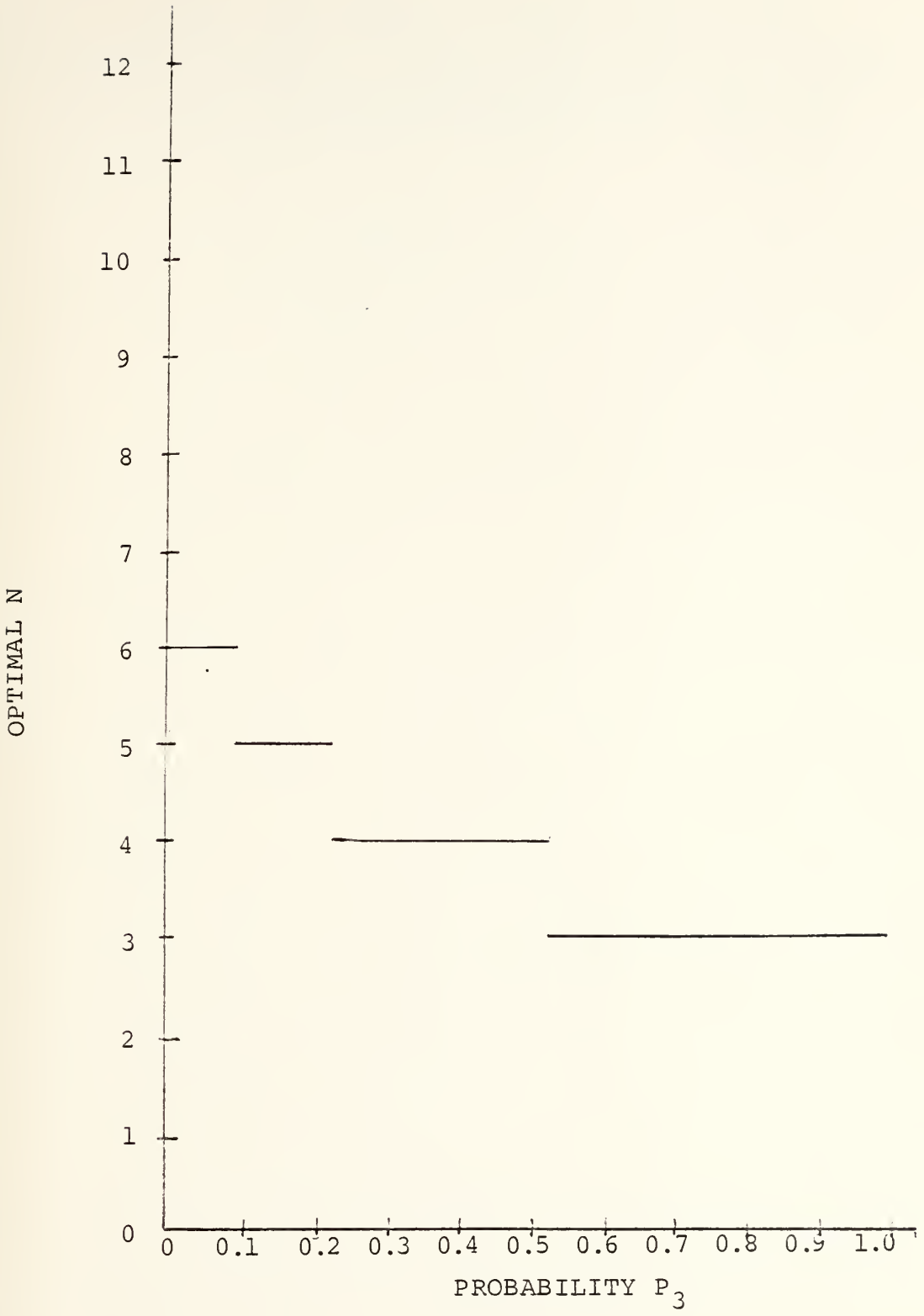

Figure 3.17 Optimal N for Three Production Lines When $C_{D_1} = C_{D_2} = C_{D_3} = \$10$ and $p_1 = p_2 = 0.1$

Figure 3.17 looks like Figure 3.11 moved to the left. Interestingly, the shift in break points is approximately 0.1, which is the demand probability of the added line.

IV. SUMMARY AND CONCLUSIONS

A. SUMMARY

A model has been developed describing the expected costs of delivery and delay per period when demand for a certain repair part comes from more than one production line at a NARF. This model extends the earlier work by McMasters and Davidson on a scheduled delivery model. Several assumptions were made to facilitate the development. First, the time periods between component inductions for each production line were considered equal. Next the transportation costs were fixed. Thirdly, the time dependent delay costs were allowed to be different for each production line. The probabilities of demand for each production line were also allowed to be different.

The objective of the model development was the determination of the optimal number of periods between deliveries. This is that number N which minimizes the expected costs of delivery and delay per period.

Unfortunately, no simple closed form expression for optimal N as a function of the other parameters could be obtained. As a consequence, parametric analyses of the cost function were conducted to determine optimal N and its behavior when other parameters were varied.

B. CONCLUSIONS

The optimization analysis showed that as the value of C_{D_ℓ} was increased for each production line, while holding the number of lines l and p_ℓ constant, the value for optimal N decreased. When the value of C_D is held constant for one production line and increased for another under the above conditions the optimal N values also decreased. The decrease for the latter case is not as severe as when all C_{D_ℓ} values are increased.

When the probability of demand is increased, holding L and C_{D_ℓ} constant, optimal N decreases. For small probabilities of demand it is less likely that repair items will be required, thereby extending the number of periods between deliveries. However, as the probabilities increase it is more likely that a demand will be made in the earlier periods.

As the number of production lines increase, the optimal value of N also decreases. Here again the decrease corresponds to the fact that the likelihood of a demand from the system has increased.

This study assumed all demands were treated equally by the NARF. In reality, this is not the case. Each demand submitted has a priority attached to it; however, within a given production line, a repair part usually has the same priority each time it is requested. Thus, to illustrate the impact of this priority, it is only necessary to change a production line's C_D value to a higher value to reflect its

increased importance. At present the priority system is an implicit way to distinguish between the fact that different lines have different C_D values because the NARF cannot compute actual C_D values.

This thesis was only able to explore the parametric influences to a limited extent. Additional analyses suggest themselves. For example, why do the break points in Figure 3.17 appear to have shifted by approximately 0.1 from those in Figure 3.11 when one production line was added. The shift of 0.1 was the same as the probability of demand for the added production line.

Perhaps more important is the need to evaluate the range of parameter values for which N = 1 since that corresponds to a delivery as soon as a demand occurs. This compares to the current twice daily delivery service provided by NSC Oakland to NARF Alameda. The results of such an analysis could be used to determine if current actual parameter values were comparable or not. If not, then perhaps less frequent service is worth considering.

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